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# A New Look at Hadronic B Decays

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# A new look at hadronic B decays

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We present a detailed study of non-leptonic two-body decays of $B$ mesons based on a generalized factorization hypothesis. We discuss the structure of non-factorizable corrections and present arguments in favour of a simple phenomenological description of their effects. We discuss tests of the factorization hypothesis and show how it may be used to determine unknown decay constants. In particular, we obtain $f_{D_s} = (234 \pm 25)$ MeV and $f_{D_s^*} = (271 \pm 33)$ MeV.

## 1. INTRODUCTION

The weak decays of hadrons containing a heavy quark offer the most direct way to determine the weak mixing angles of the Cabibbo-Kobayashi-Maskawa matrix and to explore the physics of CP violation. However, an understanding of the connection between quark and hadron properties is a necessary prerequisite for a quantitative theoretical description of these processes. The complexity of strong-interaction effects increases with the number of quarks appearing in the final state. Bound-state effects in leptonic decays can be lumped into a single parameter (the "decay constant"), while those in semileptonic decays are described by invariant form factors, depending on the momentum transfer $q^2$ between the hadrons. Approximate symmetries of the strong interactions help us to constrain the properties of these form factors [1]. For non-leptonic decays, on the other hand, we are still lacking a comprehensive understanding of strong-interaction effects even in simple decay modes. The problem is exemplified in Fig. 1, which shows multiple exchanges of gluons between the quarks in the initial and final states. The intricate interplay between weak and strong forces has many surprising consequences, whose understanding is a challenge to theory. Examples are the $\Delta I = \frac{1}{2}$ selection rule in $K$ decays, the difference in the lifetimes of the charm mesons $D^+$ and $D^0$, and the difference of the lifetimes of the $\Lambda_b$ and $B$ particles. Although strong-interaction are more dramatic at low energies, they are still hard to understand even in

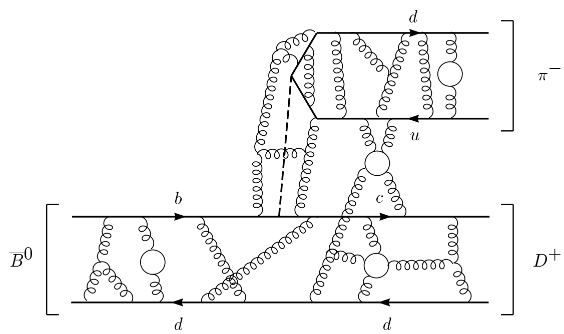


Figure 1. Strong-interaction effects in a non-leptonic decay.

$B$ decays.

At tree level in the Standard Model, non-leptonic weak decays are mediated by a single $W$-exchange diagram. When the external quarks have energies much below the electroweak scale, this process can be described by a local four-fermion interaction, which gets modified if hard gluon exchanges between the quarks are included. Their effects can be taken into account by using the renormalization group to evolve the effective interaction from the electroweak scale down to a scale $\mu$ of order $m_b$, the mass of the decaying $b$ quark. For the case of $b \to c\bar{u}d$ transitions, e.g., the relevant part of the effective Hamiltonian is

$$\frac{G_F}{\sqrt{2}}\,V_{cb}V_{ud}^*\Big\{c_1(\mu)\,(\bar{d}u)(\bar{c}b) + c_2(\mu)\,(\bar{c}u)(\bar{d}b)\Big\}\,, \quad (1)$$

where $(\bar{d}u) = \bar{d}\gamma^\mu(1-\gamma_5)u$ etc. are left-handed, colour-singlet quark currents. The Wilson coefficients $c_i(\mu)$ are known to next-to-leading order [2,3]. At the scale $\mu = m_b$, they have the values $c_1(m_b) \approx 1.13$ and $c_2(\mu) \approx -0.29$. These coefficients take into account the short-distance corrections arising from the exchange of hard gluons. The effects of soft gluons (with virtualities below the scale $\mu$) remain in the hadronic matrix elements of the local four-quark operators. A reliable field-theoretic calculation of these matrix elements is the obstacle to a quantitative theory of hadronic weak decays.

Using Fierz identities, the four-quark operators in the effective Hamiltonian may be rewritten in various forms. It is particularly convenient to rearrange them in such a way that the flavour quantum numbers of one of the quark currents match those of one of the hadrons in the final state of the considered decay process. As an example, consider the decays $B \to D\pi$. Omitting common factors, the various decay amplitudes may be written as

$$\begin{aligned} A_{B^0\to D^+\pi^-} &= \left(c_1 + \frac{c_2}{N_c}\right)\langle D^+\pi^-|(\bar d u)(\bar c b)|B^0\rangle \\ &\quad + c_2\,\langle D^+\pi^-|\tfrac{1}{2}(\bar d t_a u)(\bar c t_a b)|B^0\rangle\,, \\ A_{B^0\to D^0\pi^0} &= \left(c_2 + \frac{c_1}{N_c}\right)\langle D^0\pi^0|(\bar c u)(\bar d b)|B^0\rangle \\ &\quad + c_1\,\langle D^0\pi^0|\tfrac{1}{2}(\bar c t_a u)(\bar d t_a b)|B^0\rangle\,, \\ A_{B^-\to D^0\pi^-} &= A_{B^0\to D^+\pi^-} - \sqrt{2}\,A_{B^0\to D^0\pi^0}\,, \end{aligned} \tag{2}$$

where $t_a$ are the SU(3) colour matrices. The last relation follows from isospin symmetry of the strong interactions. The three classes of decays shown above are referred to as class-1, class-2 and class-3, respectively [4].

## 2. FACTORIZATION HYPOTHESIS

The above decay amplitudes contain the "factorizable contributions"

$$\begin{aligned} \mathcal{F}_{(BD)\pi} &\equiv \langle\pi^-|(\bar d u)|0\rangle\,\langle D^+|(\bar c b)|B^0\rangle\,, \\ \mathcal{F}_{(B\pi)D} &\equiv \langle D^0|(\bar c u)|0\rangle\,\langle\pi^-|(\bar d b)|B^0\rangle\,. \end{aligned} \tag{3}$$

The matrix elements in this equation are known in terms of the meson decay constants $f_\pi$ and $f_D$, and the transition form factors for the decays $B \to D$ and $B \to \pi$, respectively. Most of these quantities are accessible experimentally. Of course, the matrix elements appearing in (2) also contain other, non-factorizable contributions. In general, we may define process-dependent hadronic parameters $\varepsilon_1$ and $\varepsilon_8$, which contain the non-factorizable corrections, in such a way that the decay amplitudes take the form

$$\begin{aligned} A(B^0 \to D^+\pi^-) &= a_1\,\mathcal{F}_{(BD)\pi}\,, \\ A(B^0 \to D^0\pi^0) &= a_2\,\mathcal{F}_{(B\pi)D}\,, \end{aligned} \tag{4}$$

with [5]–[7]

$$\begin{aligned} a_1 &= \left(c_1(\mu) + \frac{c_2(\mu)}{N_c}\right)\left[1 + \varepsilon_1^{(BD)\pi}(\mu)\right] \\ &\quad + c_2(\mu)\,\varepsilon_8^{(BD)\pi}(\mu)\,, \\ a_2 &= \left(c_2(\mu) + \frac{c_1(\mu)}{N_c}\right)\left[1 + \varepsilon_1^{(B\pi)D}(\mu)\right] \\ &\quad + c_1(\mu)\,\varepsilon_8^{(B\pi)D}(\mu)\,. \end{aligned} \tag{5}$$

We stress that these expressions are exact; they are just a way to parametrize the relevant matrix elements of four-quark operators. The effective coefficients $a_i$ take into account all contributions to the matrix elements and are thus $\mu$ independent. The scale dependence of the Wilson coefficients is exactly balanced by that of the hadronic parameters.

Additional insight can be gained by combining these results with the $1/N_c$ expansion [8]. At the scale $\mu = O(m_b)$, the large-$N_c$ counting rules of QCD imply $c_1 = 1 + O(1/N_c^2)$, $c_2 = O(1/N_c)$, $\varepsilon_1 = O(1/N_c^2)$, and $\varepsilon_8 = O(1/N_c)$. (For scales much lower than $m_b$, the counting rules for the Wilson coefficients $c_i(\mu)$ are spoiled by large logarithms.) Hence, we expect that $|\varepsilon_1| \ll 1$, whereas contributions from $\varepsilon_8$ can be larger. Using these results, we find from (5)

$$\begin{aligned} a_1 &= c_1(m_b) + O(1/N_c^2)\,, \\ a_2 &= c_2(\mu) + c_1(\mu)\left(\frac{1}{N_c} + \varepsilon_8^{(B\pi)D}(\mu)\right) + O(1/N_c^3) \\ &\equiv c_2(m_b) + \zeta\,c_1(m_b)\,, \end{aligned} \tag{6}$$

where $\zeta = 1/N_c + \varepsilon_8(m_b)$ is a process-dependent hadronic parameter of order $1/N_c$ [7]. It is important to stress that the naive choice $a_1 = c_1 + c_2/N_c$ and $a_2 = c_2 + c_1/N_c$, which is often referred to as "factorization hypothesis", does not correspond to *any* consistent limit of QCD; in particular, this is not a prediction of the $1/N_c$ expansion. The more general expression for $a_2$ given above was first introduced in Ref. [4]. Since the parameter $\varepsilon_8$ is of order $1/N_c$, the two contributions to $\zeta$ are of the same magnitude, and hence $\zeta$ should be considered as an unknown dynamical parameter. As a general rule, we expect that non-factorizable corrections are small in class-1 transitions. In class-2 decays, on the other hand, the contribution proportional to $\varepsilon_8$ is enhanced by the large value of the ratio $c_1/c_2 = O(N_c)$, and non-factorizable contributions can therefore be sizeable.

The parameter $\varepsilon_8$ obeys the renormalization-group equation

$$\mu \frac{d}{d\mu}\, \varepsilon_8^P(\mu) \approx -\frac{4\alpha_s}{3\pi} \,, \tag{7}$$

where the superscript $P$ represents the dependence on the decay process. Let us assume that, for each process, there exists a "factorization scale" $\mu_f$ such that $\varepsilon_8^P(\mu_f) = 0$. (We will see later that this is indeed the case for two-body decays of $B$ mesons.) Then (7) implies that the phenomenological parameter $\zeta$ is given by [7]

$$\zeta \approx \frac{1}{N_c} - \frac{4\alpha_s}{3\pi}\, \ln \frac{m_b}{\mu_f} \,. \tag{8}$$

Based on the colour transparency argument of Bjorken [9], we expect that $\mu_f$ scales with the energy of the outgoing hadrons in a decay process. A fast-moving pair of quarks in a colour-singlet state acts as a colour dipole and decouples from soft gluons. Only hard gluons, with virtualities of order the energy of the outgoing particles, can rearrange the quarks and thus spoil factorization. On a qualitative level, the connection between the factorization scale and the energy release in the final state can be seen from Fig. 2, where we show the ratio $a_2/a_1$ as a function of $\alpha_s(\mu_f)$. As we will see later, the value preferred by $B \to D\pi$ decays is positive and corresponds to a rather small coupling, indicating $\mu_f = O(m_b)$ for these processes. On the other hand, $D$ decays indicate a negative value of $a_2/a_1$, corresponding to a lower value of the factorization scale. This is in accordance with the fact that in these processes the energy released to the final-state particles is much smaller.

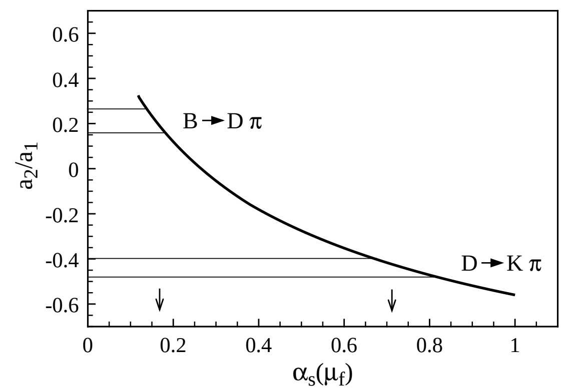


Figure 2. The ratio $a_2/a_1$ as a function of the running coupling constant evaluated at the factorization scale. The bands indicate the phenomenological values of $a_2/a_1$ extracted from $B \to D\pi$ and $D \to K\pi$ decays.

In two-body decays of $B$ mesons, the energy release (per particle) in different processes differs by less than about 1 GeV. Combining this observation with the empirical fact that in $B \to D\pi$ decays the scale $\mu_f$ is of order $m_b$, we find that the changes of the phenomenological parameter $\zeta$ in different decay channels are of order [7]

$$\Delta\zeta \sim \frac{4\alpha_s}{3\pi}\, \frac{\Delta\mu_f}{m_b} \sim \text{few }\% \,. \tag{9}$$

Thus, on rather general grounds, we expect that in two-body $B$ decays, to a good approximation, $\zeta$ is process independent. We shall refer to this assumption, together with the prediction for the parameters $a_1$ and $a_2$ given in (6), as *generalized factorization hypothesis.*

## 3. TESTS AND PREDICTIONS

Adopting the theoretical framework described above, a prediction of hadronic decay amplitudes needs as input information meson decay constants and form factors. The decay constants of many light mesons are known with high accuracy from hadronic $\tau$ decays, and from the electromagnetic decays of vector mesons [7]. The meson form factors of quark currents can, to some extend, be extracted from semileptonic decays. Whereas the various $B \to D^{(*)}$ transition form factors are well-known by combining data from semileptonic decays with heavy-quark symmetry relations [1], the available information about form factors describing the decays of $B$ mesons into light mesons is largely model dependent. As a consequence, for all class-1 decays considered below the factorized decay amplitudes can be predicted without any model assumptions; however, the theoretical predictions for class-2 amplitudes involve larger theoretical uncertainties. In this work, we will adopt the NRSX model [10] to calculate the meson form factors. Because of lack of information, we shall neglect final-state interactions between the produced hadrons in the final state. In energetic two-body decays of $B$ mesons, these effects are expected to be small [7,11].

Our predictions for the branching ratios of some of the dominant non-leptonic two-body decays of $B$ mesons are given in Table 1. The QCD coefficients $a_1$ and $a_2$, as well as the (poorly known) decay constants of charm mesons, have been left as parameters in the theoretical expressions. For comparison, we show the world average experimental results for the branching ratios, as recently compiled in the review article in Ref. [12].

### 3.1. Extractions of $a_1$

There are various ways in which to test the generalized factorization hypothesis. The most direct one relies on the close relationship between semileptonic and factorized hadronic decay amplitudes. Comparing the non-leptonic decay rates with the corresponding differential semileptonic decay rates evaluated at the same value of $q^2$ provides a direct test of the factorization hypothesis [9]. We have

$$\frac{\Gamma(B^0 \to D^{(*)+} h^-)}{d\Gamma(B^0 \to D^{(*)} \ell\,\bar\nu)/dq^2\Big|_{q^2=m_h^2}} = 6\pi^2 |V_{ud}|^2 f_h^2 a_1^2\,, \tag{10}$$

where $h$ denotes a light meson ($h = \pi$ or $\varrho$), and $f_h$ its decay constant. To determine this ratio experimentally, one needs the values of the differential semileptonic decay rate at various values of $q^2$. They have been determined for $B \to D^* \ell\,\bar\nu$ decays in Ref. [12], using a fit to experimental data. A comparison of the theoretical prediction with experimental data yields $a_1 = 1.11 \pm 0.10$ for $h = \pi$, and $a_1 = 1.09 \pm 0.13$ for $h = \varrho$.

A more precise determination of the parameter $a_1$ is obtained by comparing the theoretical predictions for the decays $B^0 \to D^{(*)+} h^-$ (with $h = \pi$ or $\varrho$) directly with the data. For these processes, the transition form factors and decay constants are well known experimentally. Therefore, the theoretical uncertainties are minimal. From a fit to the data, we obtain $a_1 = 1.08 \pm 0.04$.

An interesting alternative extraction of $a_1$ is obtained from the class of decays $B^0 \to D^{(*)+} D_s^{(*)-}$, which involve the same $B \to D^{(*)}$ transition form factors, but have a quite different kinematics since there is less energy release to the final-state particles. Unfortunately, the decay constants of $D_s$ mesons are not well known experimentally. From a fit to the data, we find $a_1 = 1.10 \pm 0.07 \pm 0.17$, where the last error takes into account the uncertainty in the decay constants.

The different determinations of the parameter $a_1$ agree well with each other and yield a result that confirms the theoretical expectation that $a_1 \approx c_1(m_b) \approx 1.13$. Within the experimental errors, there is no evidence for a process dependence of $a_1$.

### 3.2. Extractions of $a_2$

A determination of the parameter $a_2$, as well as of the relative sign between $a_2$ and $a_1$, is obtained by comparing the theoretical predictions for the decays $B^- \to D^{(*)0} h^-$ (with $h = \pi$ or $\varrho$) with the data. Since the contributions to the decay amplitudes proportional to $a_2$ involve the $B \to h$ form factors, they cannot be predicted in a model-independent way. Using the NRSX model for these form factors and assigning a conservative error, we find $a_2/a_1 = 0.21 \pm 0.05 \pm 0.04$, where the second error accounts for the model dependence. Combining this with the value for $a_1$ determined above gives $a_2 = 0.23 \pm 0.05 \pm 0.04$. The fact that the ratio $a_2/a_1$ is positive in $B$ decays implies that class-1 and class-2 decay amplitudes interfere constructively. This is in contrast with the situation encountered in charm decays,

Table 1
Theoretical predictions for the branching ratios (in %) of non-leptonic $B$ decays. In the third column, the factors containing not well known decay constants are suppressed.

| $B^0$ Modes | NRSX Model | $a_1 = 1.08$ $a_2 = 0.21$ | Experiment |
|---|---|---|---|
| Class-1 | | | |
| $D^+\pi^-$ | $0.257\ a_1^2$ | 0.30 | $0.31 \pm 0.04 \pm 0.02$ |
| $D^+\varrho^-$ | $0.643\ a_1^2$ | 0.75 | $0.84 \pm 0.16 \pm 0.07$ |
| $D^{*+}\pi^-$ | $0.247\ a_1^2$ | 0.29 | $0.28 \pm 0.04 \pm 0.01$ |
| $D^{*+}\varrho^-$ | $0.727\ a_1^2$ | 0.85 | $0.73 \pm 0.15 \pm 0.03$ |
| $D^+D_s^-$ | $0.879\ a_1^2\,(f_{D_s}/240)^2$ | 1.03 | $0.74 \pm 0.22 \pm 0.18$ |
| $D^+D_s^{*-}$ | $0.817\ a_1^2\,(f_{D_s^*}/275)^2$ | 0.95 | $1.14 \pm 0.42 \pm 0.28$ |
| $D^{*+}D_s^-$ | $0.597\ a_1^2\,(f_{D_s}/240)^2$ | 0.70 | $0.94 \pm 0.24 \pm 0.23$ |
| $D^{*+}D_s^{*-}$ | $2.097\ a_1^2\,(f_{D_s^*}/275)^2$ | 2.45 | $2.00 \pm 0.54 \pm 0.49$ |
| Class-2 | | | |
| $\bar{K}^0 J/\psi$ | $2.262\ a_2^2$ | 0.10 | $0.075 \pm 0.021$ |
| $\bar{K}^0\psi(2S)$ | $1.051\ a_2^2$ | 0.05 | $< 0.08$ |
| $\bar{K}^{*0} J/\psi$ | $3.645\ a_2^2$ | 0.16 | $0.153 \pm 0.028$ |
| $\bar{K}^{*0}\psi(2S)$ | $1.939\ a_2^2$ | 0.09 | $0.151 \pm 0.091$ |

| $B^-$ Modes | NRSX Model | $a_1 = 1.08$ $a_2 = 0.21$ | Experiment |
|---|---|---|---|
| Class-1 | | | |
| $D^0D_s^-$ | $0.938\ a_1^2\,(f_{D_s}/240)^2$ | 1.09 | $1.36 \pm 0.28 \pm 0.33$ |
| $D^0D_s^{*-}$ | $0.873\ a_1^2\,(f_{D_s^*}/275)^2$ | 1.02 | $0.94 \pm 0.31 \pm 0.23$ |
| $D^{*0}D_s^-$ | $0.639\ a_1^2\,(f_{D_s}/240)^2$ | 0.75 | $1.18 \pm 0.36 \pm 0.29$ |
| $D^{*0}D_s^{*-}$ | $2.235\ a_1^2\,(f_{D_s^*}/275)^2$ | 2.61 | $2.70 \pm 0.81 \pm 0.66$ |
| Class-2 | | | |
| $K^- J/\psi$ | $2.411\ a_2^2$ | 0.11 | $0.102 \pm 0.014$ |
| $K^-\psi(2S)$ | $1.122\ a_2^2$ | 0.05 | $0.070 \pm 0.024$ |
| $K^{*-} J/\psi$ | $3.886\ a_2^2$ | 0.17 | $0.174 \pm 0.047$ |
| $K^{*-}\psi(2S)$ | $2.070\ a_2^2$ | 0.09 | $< 0.30$ |
| Class-3 | | | |
| $D^0\pi^-$ | $0.274\,[a_1 + 1.127\,a_2\,(f_D/200)]^2$ | 0.48 | $0.50 \pm 0.05 \pm 0.02$ |
| $D^0\varrho^-$ | $0.686\,[a_1 + 0.587\,a_2\,(f_D/200)]^2$ | 0.99 | $1.37 \pm 0.18 \pm 0.05$ |
| $D^{*0}\pi^-$ | $0.264\,[a_1 + 1.361\,a_2\,(f_{D^*}/230)]^2$ | 0.49 | $0.52 \pm 0.08 \pm 0.02$ |
| $D^{*0}\varrho^-$ | $0.775\,[a_1^2 + 0.661\,a_2^2\,(f_{D^*}/230)^2 + 1.518\,a_1 a_2\,(f_{D^*}/230)]$ | 1.19 | $1.51 \pm 0.30 \pm 0.02$ |

where a similar analysis yields $a_1 = 1.10 \pm 0.05$ and $a_2 = -0.49 \pm 0.04$ [10], indicating a strong destructive interference. Since most $D$ decays are (quasi) two-body transitions, this effect is responsible for the observed lifetime difference between $D^+$ and $D^0$ mesons: $\tau(D^+) > \tau(D^0)$. In $B$ decays, on the other hand, the majority of transitions proceeds into multi-body final states, and moreover there are many $B^-$ decays (such involving two charm quarks in the final state) where no interference can occur. The relevant scale for multi-body decay modes may be significantly lower than $m_b$, leading to destructive interference (see Fig. 2). Therefore, the observed constructive interference in the two-body modes is not in conflict with the fact that $\tau(B^-) > \tau(B^0)$.

An alternative determination of the magnitude (but not the sign) of $a_2$ can be obtained from the class of decays $B \to K^{(*)}\psi^{(\prime)}$, which are characterized by a quite different decay kinematics. We find $a_2 = 0.21 \pm 0.01 \pm 0.04$. A comparison of this result with the value of $a_2$ determined above provides an interesting test of our theoretical prediction that even in decay modes with different energy release the process dependence of $a_2$ is expected to be mild. Within errors, there is indeed no evidence for any process dependence. Hence, the data fully support the generalized factorization hypothesis. From the result for $a_2$ we may then extract the value of the phenomenological parameter $\zeta$ or, equivalently, of the colour-octet matrix element $\varepsilon_8$. We obtain

$$\zeta = 0.45 \pm 0.05\,, \quad \varepsilon_8(m_b) = 0.12 \pm 0.05\,. \tag{11}$$

It would be most interesting to derive these results from a rigorous, field-theoretical evaluation of the four-quark operator matrix elements.

### 3.3. Determination of decay constants

As an application, we shall employ the generalized factorization hypothesis to obtain rather precise values for the decay constants of the $D_s$ and $D_s^*$ mesons. To this end, we derive from Table 1 the theoretical predictions for the following ratios of decay rates:

$$\frac{\Gamma(B^0 \to D^+ D_s^-)}{\Gamma(B^0 \to D^+ \pi^-)} = 1.01 \left(\frac{f_{D_s}}{f_\pi}\right)^2 ,$$

$$\frac{\Gamma(B^0 \to D^{*+} D_s^-)}{\Gamma(B^0 \to D^{*+} \pi^-)} = 0.72 \left(\frac{f_{D_s}}{f_\pi}\right)^2 ,$$

$$\frac{\Gamma(B^0 \to D^+ D_s^{*-})}{\Gamma(B^0 \to D^+ \varrho^-)} = 0.74 \left(\frac{f_{D_s^*}}{f_\varrho}\right)^2 ,$$

$$\frac{\Gamma(B^0 \to D^{*+} D_s^{*-})}{\Gamma(B^0 \to D^{*+} \varrho^-)} = 1.68 \left(\frac{f_{D_s^*}}{f_\varrho}\right)^2 . \tag{12}$$

Theoretically, these predictions are rather clean for the following reasons: first, all decays involve class-1 transitions, so that deviations from factorization are probably very small; secondly, the parameter $a_1$ cancels in the ratios; thirdly, the two processes in each ratio have a similar kinematics, so that the corresponding decay rates are sensitive to the same form factors, however evaluated at different $q^2$ values; finally, we may hope that also some of the experimental systematic errors cancel in the ratios (however, we do not assume this in quoting errors below). Combining these predictions with the average experimental branching ratios [12],

$$\begin{aligned} \mathrm{B}(B \to D D_s^-) &= (0.95 \pm 0.24)\%\,, \\ \mathrm{B}(B \to D D_s^{*-}) &= (1.00 \pm 0.30)\%\,, \\ \mathrm{B}(B \to D^* D_s^-) &= (1.03 \pm 0.27)\%\,, \\ \mathrm{B}(B \to D^* D_s^{*-}) &= (2.26 \pm 0.60)\%\,, \end{aligned} \tag{13}$$

we find the rather accurate values

$$\begin{aligned} f_{D_s} &= (234 \pm 25)\ \mathrm{MeV}\,, \\ f_{D_s^*} &= (271 \pm 33)\ \mathrm{MeV}\,. \end{aligned} \tag{14}$$

The result for $f_{D_s}$ is in excellent agreement with the value $f_{D_s} = 241 \pm 37$ MeV extracted from the leptonic decay $D_s \to \mu^+\nu$ [14]. The ratio $f_{D_s^*}/f_{D_s} = 1.16 \pm 0.19$, which cannot be determined from leptonic decays, is in good agreement with theoretical expectations [15,16]. Finally, we note that, assuming SU(3) breaking effects of order 10–20%, the established value of $f_{D_s}$ implies $f_D \approx 200$ MeV, which is larger than most theoretical predictions.

## 4. SUMMARY

Exclusive hadronic decays of $B$ mesons are strongly influenced by the long-range QCD colour forces. Theoretically, their description involves hadronic matrix elements of local four-quark operators, which are notoriously difficult to calculate. The factorization approximation is used to relate these matrix elements to products of current matrix elements. Conventionally, the factorized decay amplitudes depend on two phenomenological parameters $a_1$ and $a_2$, which are connected with the Wilson coefficients $c_i(\mu)$ appearing in the effective weak Hamiltonian. We have shown that this approach can be generalized in a natural way to include the dominant non-factorizable contributions to the decay amplitudes. (The situation is more complicated in $B$ decays into two vector mesons, see Ref. [7].) In the generalized factorization scheme, the effective parameters $a_1$ and $a_2$ become process-dependent. However, using the large-$N_c$ counting rules of QCD, we have argued that in energetic two-body decays of $B$ mesons $a_1 \approx c_1(m_b)$ and $a_2 \approx c_2(m_b) + \zeta\, c_1(m_b)$, where $\zeta = O(1/N_c)$ is a dynamical parameter. Moreover, the colour transparency argument suggests that the process dependence of $\zeta$ is likely to be very mild, so that it can be taken to be a constant for a wide class of two-body decays. These theoretical expectations are fully supported by the data. From a fit to the world average branching ratios of two-body decay modes, we obtain $a_1 \approx 1.08 \pm 0.04$ and $a_2 \approx 0.21 \pm 0.05$, corresponding to $\zeta \approx 0.45 \pm 0.05$. There is no evidence for a process dependence of these parameters; in particular, the values obtained for $a_2$ from the decays $B \to \bar{K}^{(*)}\psi^{(\prime)}$ and $B^- \to D^{(*)0}h^-$, where $h = \pi$ or $\varrho$, are in good agreement with each other.

We have discussed various tests of the generalized factorization hypothesis by considering ratios of decay rates, and by comparing non-leptonic decay rates with semileptonic rates evaluated at the same value of $q^2$. Within the present experimental uncertainties, there are no indications for any deviations from the factorization scheme in which $a_1$ and $a_2$ are treated as process-independent hadronic parameters. Accepting that this scheme provides a useful phenomenological concept, exclusive two-body decays of $B$ mesons offer a unique opportunity to measure the decay constants of some light or charm mesons, such as the $a_1$, $D_s$ and $D_s^*$.

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